Math 141 **Summer 2000**

Practice Midterm

The actual midterm will be much shorter. No calculators will be allowed.

1. Sketch the graphs of the following functions, where a and b are positive constants.
   (a) $y = x^3$
   (b) $y = (x-a)^3$
   (c) $y = x^3 + b$
   (d) $y = (x-a)^3 + b$
   (e) $y = -x^3$
   (f) $y = \frac{1}{2}x^3$
2. Find the function $f(x) = ae^x + b$ whose graph is shown.
3. Sketch the graphs of the following functions:
   (a) $y = \ln x$
   (b) $y = \ln(-x)$
   (c) $y = \ln|x|$
   (d) Find a formula for the function whose graph is shown

4. Sketch the graph of an example of a function $f(x)$ that satisfies all the given conditions:

$$\lim_{x\to-\infty} f(x) = 0, \quad f(-1) = 1, \quad \lim_{x\to 0^-} f(x) = +\infty,$$

$$f(0) \textit{ is not defined}, \quad \lim_{x\to 0^+} f(x) = -\infty, \quad \lim_{x\to 1^-} f(x) = -1,$$

$$f(1) = 0, \quad \lim_{x\to 1^+} f(x) = 1, \quad \lim_{x\to+\infty} f(x) = 2.$$

5. Find an equation of the tangent line to the curve $y = x^2 - 1$ at the point (1,0).

6. Find the derivatives of the given functions using the definition of derivative.

   (a) $f(x) = 1$
   (b) $f(x) = x$
   (c) $f(x) = x^2$
   (d) $f(x) = x^3$

7. Find the following limits:

   (a)
   $$\lim_{x\to 5} (7x - 25)$$

   (b)
   $$\lim_{h\to 0} \frac{(1+h)^2 - 1}{h}$$

   (c)
   $$\lim_{t\to 2} \frac{3}{(t-2)^3}$$

   (d)
   $$\lim_{x\to 0} \frac{1 - \sqrt{1 - x^2}}{x}$$

   (e)
   $$\lim_{x\to -1} \frac{x^2 - x - 2}{x^2 + 3x + 2}$$

8. Find all values of a such that f is continous on $\mathbb{R}$:

$$f(x) = \begin{cases} 3x - 2 & if \quad x \leq a \\ x^2 & if \quad x > a \end{cases}$$

9. Sketch the graphs of
   (a) $y = \sin x$
   (b) $y = \cos x$.

10. Solve the equations
    (a) $\sin x = \cos x$
    (b) $\sin x = \frac{1}{2}$.

If you are having trouble with some sections or just want to practice more, I recommend you to look at the following exercises:

- **1.3** 9, 13, 15, 17, 19
- **1.5** 7, 13
- **1.6** 47, 48
- **2.2** 5, 21
- **2.3** 3, 5, 10, 11, 15, 27, 39
- **2.4** 15
- **2.5** 35, 37
- **2.6** 5, 7, 19, 27, 42, 46
- **2.7** 7, 9, 12
- **2.8** 5, 7, 15, 19, 21
- **2.9** 19, 25

Please feel free to ask any questions concerning these or any other exercises.