Math 142 **Summer 2000**

Practice Midterm

The actual midterm will be much shorter. No calculators will be allowed.

1. Consider the function $f(x) = x^4 - 4x^3 + 4x^2 + 1$ defined on $[-2, +\infty)$.
   (a) Find the intervals of increase and decrease of $f(x)$.
   (b) Find the absolute and local maximum and minimum values of $f(x)$ (on the given interval).
   (c) Find the intervals of concavity and the inflection points.
   (d) Sketch the graph of $f(x)$.

2. Find the following limits:
$$\lim_{x\to 0} \frac{x^3 - 2x}{\sin x}, \qquad \lim_{x\to\infty} \frac{\ln^2 x}{x}$$

3. Draw a careful sketch of $f(x)$, if
$$\lim_{x\to-\infty} f(x) = 0,\ \lim_{x\to 2^-} f(x) = -\infty,\ \lim_{x\to 2^+} f(x) = +\infty,\ \lim_{x\to+\infty} f(x) = 0\,,$$
   f is increasing on $(-\infty, -2)$ and decreasing on $(-2, 2)$ and $(2, +\infty)$, $f(-2) = 2$, $f(0) = 0$, $f(3) = 1$, f is CU on $(-\infty, -3)$, $(-1, 0)$, and $(1, +\infty)$ and CD on $(-3, -1)$ and $(0, 1)$.

4. Show that $f(x) = ax \cdot e^{bx}$ has a horizontal asymptote $y = 0$ for any $a, b \neq 0$

5. A piece of wire 10m long is cut into 2 pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is
   (a) a maximum?
   (b) A minimum?

6. Find $f(x)$ if $f'(x) = 1 - 8x^3 + 2\sin x - \cos x + 3e^x$, $f(0) = 5$.

7. (a) Express the limit as a definite integral on the interval $[0, \pi]$:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sin(3x_i)\Delta x$$

(b) Express the integral as a limit of sums: $\int_2^{10} x^6 dx$

8. Evaluate the following integrals:

$$\int_0^1 \frac{x dx}{x^2+1}, \quad \int_{-e^2}^{-e} \frac{3dx}{x}, \quad \int \tan(4x)dx, \quad \int \cos x \cos(\sin x)dx$$

9. Use Part 1 of the Fundamental Theorem of Calculus to find the derivatives of the following functions:

$$g(x) = \int_x^2 \cos t^2 dt, \quad h(x) = \int_{-5}^{\sin x} t\cos(t^3)dt$$

If you are having trouble with some sections or just want to practice more, I recommend you to look at the following exercises:

- **4.4** 7, 9, 15, 23, 43, 51, 53, 61
- **4.5** 5, 19, 31, 45
- **4.7** 9, 27, 42, 50
- **4.10** 3, 13, 17, 19, 23, 33, 61
- **5.1** 15, 19
- **5.2** 15, 19, 27, 29, 33, 35, 43, 63
- **5.3** 5, 9, 15, 19, 33, 47, 57
- **5.4** 17, 19, 39, 41, 45, 53
- **5.5** 3, 9, 19, 29, 40, 51, 54, 63, 81

Please feel free to ask any questions concerning these or any other exercises.