

Frequency Modulation (FM)

Modules: Audio Oscillator, Wideband True RMS Meter, VCO, Utilities, Twin Pulse Generator, Tuneable LPF, Speech, Headphones

0 Pre-Laboratory Reading

Frequency modulation (FM) is an analog modulation technique for which the message signal is conveyed through variations in the frequency of the carrier. FM is used for radio broadcasting, public-safety radio (police and fire), marine radio, amateur (ham) radio, and general mobile radio service (walkie-talkie). FM is also used in radar.

0.1 Voltage-Controlled Oscillator (VCO)

A carrier with FM is of the form

$$y(t) = \cos \left[2\pi f_c t + 2\pi \Delta f \int x_n(t') dt' \right] \quad (1)$$

where f_c is the nominal carrier frequency, Δf is the frequency deviation (a parameter), and $x_n(t)$ is the normalized message signal. The instantaneous (cyclical) frequency of this carrier is found by taking the derivative of the argument of the cosine and then dividing by 2π . According to the Fundamental Theorem of Calculus,

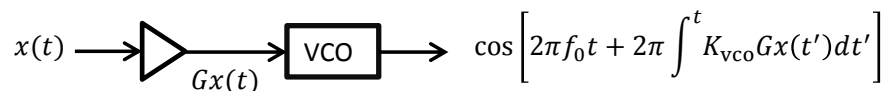
$$\frac{d}{dt} \int x_n(t') dt' = x_n(t) \quad (2)$$

Therefore,

$$\frac{1}{2\pi} \cdot \frac{d}{dt} \left[2\pi f_c t + 2\pi \Delta f \int x_n(t') dt' \right] = f_c + \Delta f x_n(t) \quad (3)$$

The instantaneous frequency for the carrier of Eq. (1) is $f_c + \Delta f x_n(t)$. In words, the instantaneous frequency is offset from the nominal carrier frequency by an amount that is proportional to the message signal.

One straightforward method for generating this signal is with a VCO.



The parameter K_{vco} has units of hertz per volt, and it represents the sensitivity of the VCO output frequency to the VCO input voltage. For the VCO used in this experiment, K_{vco} is adjustable but always negative. This means that a positive voltage at the VCO input produces a frequency at the VCO output that is less than the nominal VCO frequency f_0 . In this experiment, an amplifier with negative gain will always be used in front of the VCO so that the two minus signs cancel. A positive value for $x(t)$ will produce a positive offset in the VCO output frequency. When a VCO is used to generate a carrier, the nominal VCO frequency will equal the nominal carrier frequency. That is, $f_c = f_0$.

The maximum of the absolute value of the message signal $x(t)$ is of importance; this is denoted here by p ($p \geq 0$).

$$p = \max |x(t)| \quad (4)$$

The normalized message signal $x_n(t)$ is:

$$x_n(t) = x(t)/p \quad (5)$$

The maximum of the absolute value of $x_n(t)$ is therefore

$$\max |x_n(t)| = 1 \quad (6)$$

The expression for the VCO output equals the right-hand side of Eq. (1) when

$$\Delta f = pGK_{\text{vco}} \quad (7)$$

In this expression, p is positive. G and K_{vco} are both negative. The frequency deviation Δf is therefore positive; it represents, for a given message signal $x(t)$, the maximum deviation of the carrier instantaneous frequency from the nominal frequency. With $-p \leq x(t) \leq p$, the instantaneous carrier frequency lies, at every instant in time, between $f_c - \Delta f$ and $f_c + \Delta f$. It should be noted that Δf is in hertz, since p is in volts, G is dimensionless and K_{vco} has units hertz/volt.

0.2 Bandwidth of FM Carrier

It is important to estimate the bandwidth of a carrier with FM. This bandwidth depends, in general, on both the bandwidth W of the message signal and the frequency deviation Δf . One common estimate of the bandwidth is Carson's rule:

$$\text{Bandwidth of FM Carrier} \cong 2(\Delta f + W) \quad (8)$$

In the case where the message signal bandwidth is much larger than the frequency deviation, this bandwidth is approximately $2W$, the same bandwidth as for DSB and AM. However, for most practical FM systems $\Delta f > W$. The bandwidth of the typical FM system exceeds that for DSB and AM.

0.3 Sinusoidal Message and Bessel Functions

As usual, a sinusoidal message is employed in testing. In this case, the normalized message signal is modeled as

$$x_n(t) = \cos(2\pi f_m t) \quad (9)$$

with frequency f_m . The integral within the right-hand side of Eq. (1) becomes

$$\int_0^t x_n(t') dt' = \frac{\sin(2\pi f_m t)}{2\pi f_m} \quad (10)$$

A modulation index β for FM is defined as

$$\beta = \frac{\Delta f}{f_m} \quad (11)$$

β is dimensionless. Combining Eqs. (1), (10) and (11),

$$y(t) = \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad (12)$$

For FM, as with all analog modulation schemes, the spectrum of the modulated carrier consists of a set of spectral lines whenever the message signal is a sinusoid. For FM the spectral lines are located at the frequencies

$$f_c + k f_m, \quad k = 0, \pm 1, \pm 2, \pm 3, \dots$$

The spectral lines corresponding to positive values of k are called *upper sidebands*. The spectral lines corresponding to negative values of k are called *lower sidebands*. The spectral line at frequency f_c (corresponding to $k = 0$) is called the *carrier*. For the sake of clarity, this spectral line is also sometimes called the *residual carrier*.

The power P_k in the spectral line at $f_c + k f_m$ is given by

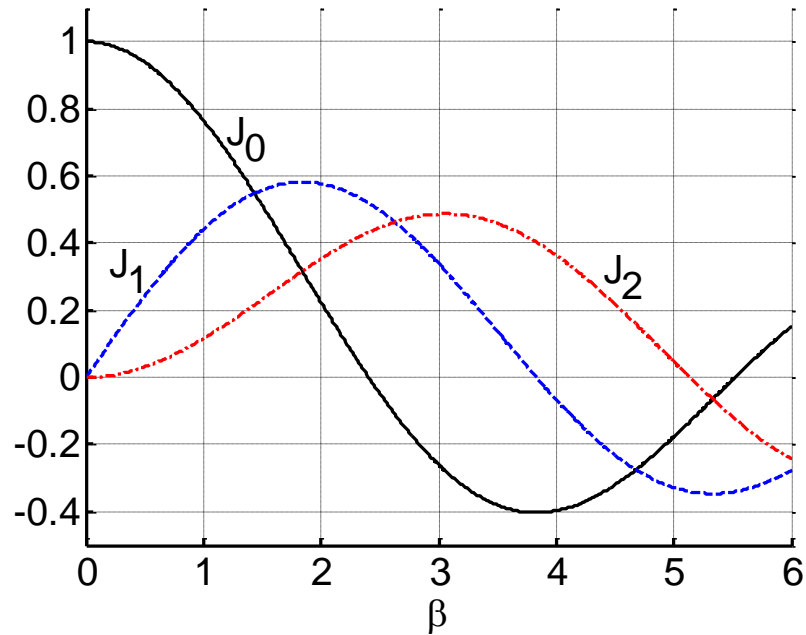
$$P_k = J_k^2(\beta) \cdot P \quad (13)$$

where P is the total power in the modulated carrier and $J_k(\cdot)$ is a Bessel function of the first kind of order k . Eq. (13) is based on a one-sided line spectrum. That is to say, all of the power in the modulated carrier is taken into account within the positive half of the frequency line.

Bessel functions can be easily computed in MATLAB and many other programs for numerical analysis. $J_k(\beta)$ can be computed like this:

$$\mathbf{besselj(k,beta)} \quad (\text{MATLAB})$$

The Bessel functions $J_0(\cdot)$, $J_1(\cdot)$ and $J_2(\cdot)$ are plotted below.



Bessel functions of the first kind of orders 0, 1, and 2

There are several properties of Bessel functions that are useful in the theory of FM. The following property, for integer order k ,

$$J_{-k}^2(\beta) = J_k^2(\beta) \quad (14)$$

means that

$$P_{-k} = P_k \quad (15)$$

In words, two sidebands that are symmetrically located about f_c (a lower sideband and an upper sideband) contain the same power. For example, the lower fundamental sideband, at $f_c - f_m$, and the upper fundamental sideband, at $f_c + f_m$, have the same power. Another property of Bessel functions is

$$\sum_{k=-\infty}^{\infty} J_k^2(\beta) = 1 \quad (16)$$

Combing this property with Eq. (13) gives

$$\sum_{-\infty}^{\infty} P_k = P \quad (17)$$

In words, the sum of the power in all sidebands (lower and upper) plus the power in the residual carrier equals the total power of the modulated carrier.

In experimental work it is useful to have expressions for the power in a spectral line relative to that in the residual carrier. In decibels, this is

$$\frac{P_k}{P_0} = 20 \log \left[\frac{J_k(\beta)}{J_0(\beta)} \right] \text{ dBc} \quad (18)$$

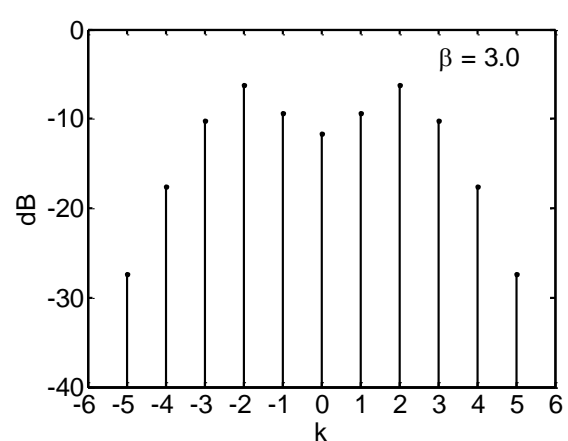
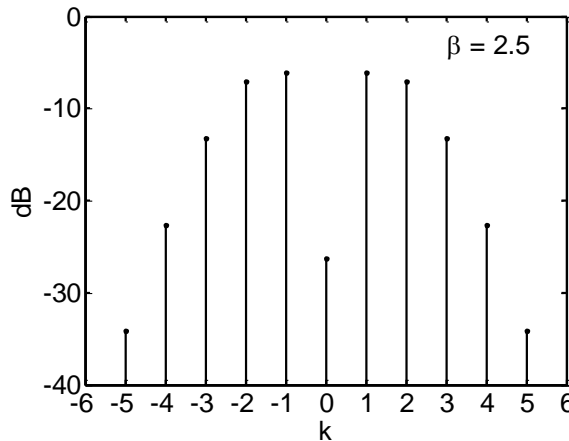
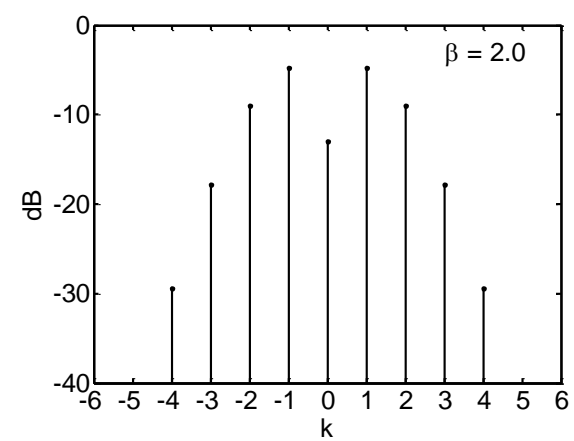
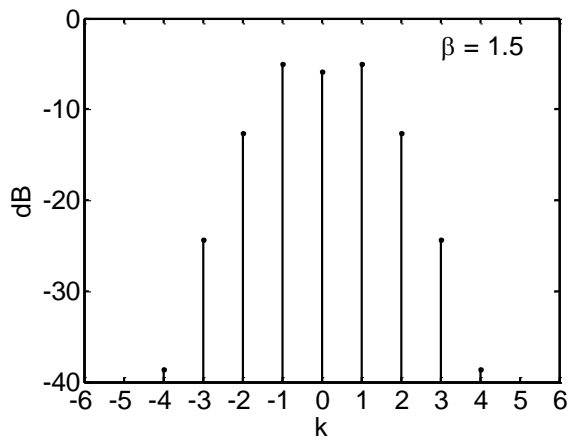
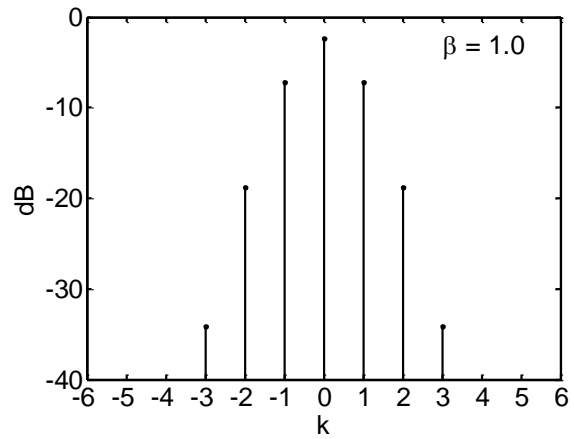
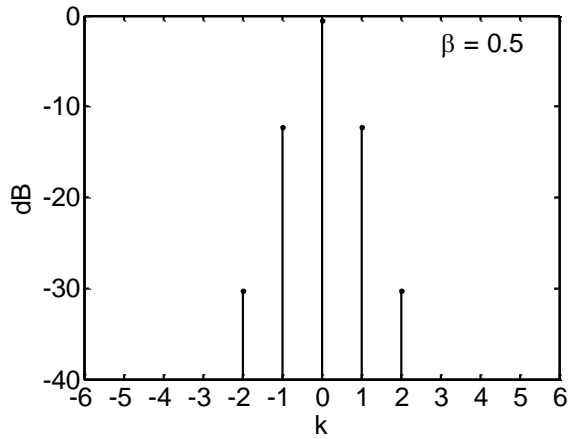
where, as usual, dBc means decibels relative to the (residual) carrier. Table 1 lists P_1/P_0 for several values of β . This table indicates, for example, that when $\beta = 0.5$, the upper fundamental and the lower fundamental will each be smaller than the line at f_c (the residual carrier) by 11.8 dB. As a second example, when $\beta = 2.0$, the upper fundamental and the lower fundamental will each be larger than the residual carrier by 8.2 dB.

Table 1: P_1/P_0 as a Function of β for a Sinusoidal Message

β	P_1/P_0 (dBc)
0.5	-11.8
1.0	-4.8
1.5	0.7
2.0	8.2
2.5	20.2
3.0	2.3

For a sinusoidal message signal, Carson's rule may be modified as follows. The message signal bandwidth W equals f_m in this case. Using Eqs. (8) and (11), the estimate of bandwidth for the modulated carrier becomes

$$\text{Bandwidth of FM Carrier} \cong 2f_m(\beta + 1) \quad (19)$$

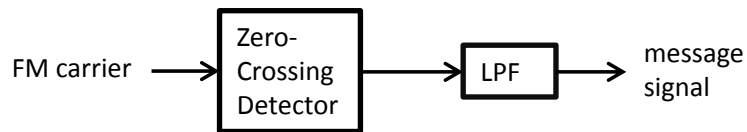


Spectrum of an FM carrier with sinusoidal message signal: P_k/P (dB) versus k

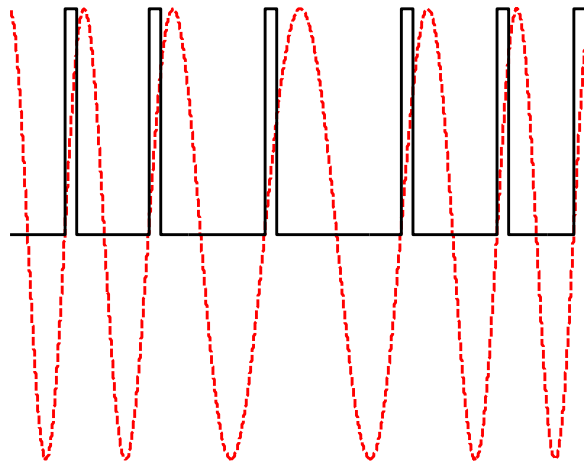
When the message signal is a sinusoid with a given frequency f_m , the number of significant spectral lines increases as the modulation index increases. The bandwidth of the modulated carrier is in approximate agreement with Carson's rule as given in Eq. (19).

0.4 Zero-Crossing Detector

One simple method of demodulating an FM carrier is by means of a zero-crossing detector.



The FM carrier enters a zero-crossing detector. This detector produces a pulse for every positive-going zero-crossing in the signal on its input. All pulses have a common height and a common width. When the instantaneous frequency of the FM carrier is relatively large, the pulses are closely spaced. When the instantaneous frequency is relatively small, the pulses are less frequent.



FM carrier (dashed) and the output of a zero-crossing detector (solid)

When the output of the zero-crossing detector is passed through a low-pass filter, the result can be a reasonable approximation of the message signal. For this to work, the bandwidth of the low-pass filter must be greater than the bandwidth of the message signal but less than the nominal carrier frequency.

1 FM with VCO

You will generate FM using a VCO. This is the direct method. You will also demonstrate that a carrier with FM can be detected with a zero-crossing detector.

1.1 Direct Method of Generating FM

Make sure the switch on the VCO module's PCB is set to "VCO". Set the toggle switch on the front panel of the VCO module to "HI". Place the VCO output on the input of the Frequency Counter. With the gain knob on the VCO module set in the fully counterclockwise (zero-gain) position, use the frequency knob to set the VCO output frequency to approximately 100 kHz. You will probably see the frequency slowly drift because the VCO does not have good frequency stability. (This is unlike the signals coming from the Master Signals module. These are synthesized from a crystal oscillator.)

In the first instance, connect the Variable DC output to the VCO input. Adjust the DC source for a positive voltage, confirming this with the oscilloscope. Rotate the gain knob on the VCO clockwise until the VCO output frequency changes significantly from the original 100 kHz. K_{VCO} for this VCO is negative, so the VCO output frequency should decrease in response to the positive input voltage.

For this experiment it is desirable to have the VCO frequency increase in response to a positive voltage. Therefore, the Variable DC output should be disconnected from the VCO and then connected through a Buffer Amplifier (having negative gain) to the VCO. You should now observe that a positive voltage on the Variable DC output produces an increase in the VCO frequency.

Adjust the Variable DC output to +2 V. Then adjust the gain of the Buffer Amplifier so that its output is -1 V. The RMS Meter can provide excellent accuracy in measuring the absolute value of these DC voltages; however, the oscilloscope is needed in order to verify the correct algebraic sign on these voltages.

Now adjust the gain on the VCO so that the VCO output frequency is approximately 1 kHz larger than its value with no input voltage. The sensitivity K_{VCO} now has the approximate value -1 kHz/V. Furthermore, the product GK_{VCO} currently has the value +500 Hz/V. In the following experimental procedure, the gain knob on the VCO will remain untouched but the Buffer Amplifier gain will be changed. K_{VCO} will retain the value -1 kHz/V. G , and therefore the product GK_{VCO} , will change.

Make measurements that will complete a table of VCO frequency offset as a function of DC voltage at the VCO input.

 VCO

The first column in the table is the DC voltage at the VCO input (not the Variable DC output). The second column is the measured VCO output frequency minus the VCO frequency in the absence of an input. Because K_{VCO} is negative, the frequency offset should be positive when the VCO input voltage is negative and vice versa. It will be necessary to adjust both the Variable DC output and the Buffer Amplifier gain in order to reach all voltages in the range -8 V to $+8\text{ V}$.



Plot the measured frequency offset as a function of input DC voltage, using discrete points. The plotted data should approximate a straight line with slope -1 kHz/V .

Replace the DC voltage from the Variable DC module with a sinusoid from the Audio Oscillator. The output of the Audio Oscillator should pass through the Buffer Amplifier to the VCO input. Adjust the frequency of the Audio Oscillator to approximately 1 kHz . Monitor the rms voltage at the VCO input. Adjust the gain of the Buffer Amplifier so that the VCO input is a sinusoid with a peak voltage of 7 V . (What is the corresponding rms voltage for this sinusoid?) Connect the VCO output to Channel A. Use the VCO output as the trigger. The display won't be stable because the signal depends on two different, unrelated frequencies: the nominal carrier frequency (set in the VCO) and the 1-kHz sinusoid (from the Audio Oscillator). Nonetheless, the display should suggest that the instantaneous frequency of the VCO output is changing in a periodic way.



Channel A: VCO output

1.2 Spectrum of FM

The VCO sensitivity K_{VCO} should be kept the same as above (-1 kHz/V). Adjust the Audio Oscillator for a frequency of approximately 1 kHz . The Audio Oscillator output should pass through the Buffer Amplifier to the VCO input (as above). Connect the VCO output to Channel A and set the PicoScope for Spectrum Mode.

You will be adjusting the voltage at the VCO input and, for each voltage adjustment, making measurements of the VCO output spectrum. The measurements, along with some calculated values, will be recorded in an Excel worksheet.



FM spectrum

For each row in the Excel worksheet table, you will set the frequency deviation with the aid of Eq. (7) by adjusting the product pG to the correct value. Please note that pG is simply the peak voltage at the input to the VCO. To help you do this, you should monitor the VCO input with the RMS Meter and take into account the relationship between an rms voltage and the corresponding peak voltage (such as pG).



Ask your instructor to verify your procedure for setting Δf .

The second and third columns of the table can be completed using Eqs. (11) and (19), respectively.

For each Δf (that is to say, for each row in the table) observe the spectrum of the VCO output. But don't change Δf until you've completed the measurements, described below, for each spectrum.



Channel A: VCO output

Capture this spectrum for each Δf . (That's a total of 6 spectra.)

For each row in the table, you should compare the spectrum with that expected from theory. You should consider whether the Carson's rule estimate of FM bandwidth, Eq. (19), is reasonable based on the observed spectrum. It will be helpful to zoom in to that part of the spectrum near the residual carrier. You can do this by selecting zoom and then changing the position and size of the Zoom Overview box.

The fourth and fifth columns in the table will contain measurement values. The fourth column is the line height of the residual carrier (in dBu). The fifth column is the line height of the upper fundamental (in dBu). The line height of the lower fundamental, which you will not record, should be approximately the same as that for the upper fundamental. A convenient and accurate method for making these measurements is to use two vertical rulers. Place one vertical ruler approximately at the frequency of the residual carrier and the second at approximately the frequency of the upper fundamental. Add two measurements to the spectrum display.

Measurements > Add Measurement

A dialog box will pop up, and you should select an *amplitude at peak* measurement for the *peak nearest ruler 1*. Repeat this procedure for a second amplitude measurement for the *peak nearest ruler 2*.

The final column of the above table will contain the upper fundamental power in dBu (column 5) minus the residual carrier power in dBu (column 4). You should compare your measurement results with the predictions (based on theory) of Table 1.



Plot the measured value of P_1/P_0 (in dBc) as a function of β , using discrete points. Also plot (on the same set of axes) the theoretical value of P_1/P_0 (in dBc) as a function of β , using discrete points. The theoretical value is given in Table 1.

1.3 Zero-Crossing Detector

A carrier with FM will now be detected using a zero-crossing detector. First, generate a carrier with FM. You can use the same set-up as above: the Audio Oscillator output passes through a Buffer Amplifier to the VCO input.

Set the toggle switch on the front panel of the VCO module to “LO” and then adjust the nominal frequency of the VCO to 10 kHz. (Yes, you heard right: only ten kilohertz.) For this demonstration, the residual carrier will have an unusually small frequency because this provides a clearer view of the detection process on the oscilloscope.

Set the VCO sensitivity K_{VCO} to approximately -1 kHz/V. Even though you had earlier adjusted the VCO gain knob to this sensitivity, you should check this. Changing the VCO nominal frequency to 10 kHz might have affected the sensitivity.

Adjust the Audio Oscillator frequency to approximately 500 Hz. Adjust the gain of the Buffer Amplifier so that the peak voltage at the VCO input is approximately 7 V.

The Twin Pulse Generator will be part of the zero-crossing detector. Make sure the slide switch on the PCB of this module is set to *single* mode. Rotate the width knob so that the line on the knob points vertically, so that the pulse width is not too narrow and not too wide.

Connect the FM carrier (the output of the VCO) to the analog signal input of the comparator on the Utilities module. The analog reference input to the comparator should be ground. Connect the comparator TTL output to the clock input of the Twin Pulse Generator. The combination of the comparator and the Twin Pulse Generator constitutes a zero-crossing detector. Connect the Q_1 output of the Twin Pulse Generator to the input of a Tuneable LPF. Adjust the filter bandwidth to approximately 1 kHz. (The Tuneable LPF’s clock output has a frequency equal to 100 times the bandwidth.)

Connect the VCO output to Channel A and the zero-crossing detector output (the Q_1 output of the Twin Pulse Generator) to Channel B. Even though it won’t be possible to get a stable display (without stopping signal capture), for best viewing you should use the VCO output as the trigger source. This oscilloscope display demonstrates the action of the zero-crossing detector.



Channel A: VCO output

Channel B: zero-crossing detector output (Q_1 output of the Twin Pulse Generator)

Connect the zero-crossing detector output to Channel A and the LPF output to Channel B. Use the TTL output from the Audio Oscillator as an external trigger source. (As always with a TTL signal, the trigger level must be positive.) This display demonstrates the action of the LPF on the pulsed signal coming from the zero-crossing detector.



Channel A: zero-crossing detector output (Q_1 output of the Twin Pulse Generator)
Channel B: LPF output

Connect the original message signal (the Audio Oscillator output) to Channel A and the LPF output to Channel B. Use the TTL output from the Audio Oscillator as an external trigger source. The LPF output should be a reasonable approximation of the original message signal. There will be delay, and the amplitude will be different. However, the LPF output should approximate a sinusoid of the correct frequency.



Channel A: Audio Oscillator output
Channel B: LPF output

1.4 FM with Speech

Change the VCO nominal frequency back to 100 kHz by setting the toggle switch back to “HI” and adjusting the frequency knob. Recalibrate the VCO sensitivity K_{vco} for -1 kHz/V.

Replace the Audio Oscillator with a speech signal.

To demodulate this FM carrier with speech signal, use the same zero-crossing detector that you built above, except make one change. On the Twin Pulse Generator, use the narrowest available pulse width. (That is, rotate the width knob fully counter-clockwise.) Connect the output of the LPF (the Tuneable LPF that forms the final stage of the zero-crossing detector) to the headphones and listen to the detected signal. Adjust the gain of the Buffer Amplifier in the modulator and the bandwidth of the LPF in the detector as necessary to produce good sound quality at the output of the detector.