

The exam will take place during the first hour of class. You will have from 4:00 - 5:00 to complete your work on the exam.

1. Make sure you are able to do all of the problems on both of the quizzes.
2. Make sure you are able to do all of the problems on all of the homeworks.

1.4 The Fibonacci Sequence

Essential Skill 1 Know the definition of the Fibonacci sequence and be able to prove basic results about it

Example Problems: Let F_n denote the n^{th} Fibonacci number.

1. Prove that $F_{n+3} + F_n = 2F_{n+2}$ for any n .
2. Prove that $F_{n+3} - F_n = 2F_{n+1}$ for any n .

Section 1.5 - Divisibility

Essential Skill 1 Know the basic definition of divisibility and be able to use theorems such as 1.8 and 1.9

Essential Skill 2 Understand the division algorithm and be able to use it

Example Problems:

1. Your homework problems from this section.
2. Prove that the sum of two even or two odd integers is even, but that the sum of an odd integer and an even integer is odd.

Section 3.1 - Prime Numbers

Essential Skill 1 Know definitions and examples of prime numbers. Be able to use the basic primality test (Theorem 3.2) to determine if an integer is prime.

Essential Skill 2 Be able to argue that there are an infinite number of prime numbers.

Example Problems:

1. Your homework problems from this section
2. Prove that the integer $n! + 1$ has a prime divisor greater than n . Conclude that there are infinitely many prime numbers.
3. Find all primes that can be written in form $n^3 + 1$ where $n \in \mathbb{N}$.

Section 3.3 - Greatest Common Divisors

Essential Skill 1 Be able to find the GCD of two integers.

Essential Skill 2 Understand the connection between the GCD and linear combinations (Theorem 3.8 and 3.9)

Example Problems:

See the Diophantine Equations Section

Essential Skill 3 Know what it means for two integers to be relatively prime.

Example Problems:

1. Your homework problems from this section
2. **True or False:** If $k \in \mathbb{N}$ then $3k + 2$ and $2k + 3$ are relatively prime. Prove true or give a counterexample.
3. **True or False:** If n is even and m is odd then n and m are relatively prime. Prove true or give a counterexample.

Section 3.4 - Euclidean Algorithm

Essential Skill 1 Be able to write the GCD of two integers as a linear combination of the two integers.

Example Problems:

1. Your homework problems from this section

Section 3.5 - Fundamental Theorem of Arithmetic

Essential Skill 1 Be able to use the prime factorization of a number to answer a variety of questions

Example Problems:

1. Your homework problems.
2. Show that $\sqrt{2} + \sqrt{3}$ is irrational.
3. Describe all integers with exactly three divisors. Same for four, five, and eight divisors.

Essential Skill 2 Know the formula for the number of times a prime occurs in the prime factorization of a factorial.

Example Problems:

1. How many divisors does $100!$ have? How many zeros does $100!$ end with?

Essential Skill 3 Understand and be able to find the LCM of two integers

Example Problems:

1. Your homework problems.
2. Show that every common multiple of a and b is divisible by $\text{lcm}(a, b)$.

Essential Skill 4 Know the GL theorem (Theorem 3.16) and be able to apply it to solve problems.

Example Problems:

Find all pairs of integers that have gcd 12 and lcm 7560.

Section 3.7 - Linear Diophantine Equations

Essential Skill 1 Know and be able to apply the Fund Theorem of Linear Diophantine Equations (Theorem 3.23). Also be able to find strictly the positive solutions to a LDE as might occur in a real-world problem

Example Problems:

See homework #4.

Section 4.1 - Congruences

Essential Skill 1 Understand what it means for two numbers to be congruent. Be able to use Theorem 4.1 (Congruence Characterization).

Essential Skill 2 Understand the Residue Theorem (Lemma 4.1). Be able to find the congruence class of a number and determine when two numbers are in the same congruence class.

Example Problems:

See homework #4.

Essential Skill 3 Be able to find least positive residues.

Example Problems:

Find the least positive residue of $1! + 2! + 3! + \cdots + 100!$ modulo 2, modulo 7, and modulo 12.

Essential Skill 4 Be able to use residues and Theorem 4.5 to simplify remainder computations.

Example Problems:

1. What is the remainder when 2^{200} is divided by 12?
2. Which decimal digits can occur in the final digit of the fourth power of an integer?

Section 4.2 - Linear Congruences

Essential Skill 1 Understand the Fundamental Theorem of Linear Congruences (Theorem 4.10) and be able to use it to find the solutions to a linear congruence.

1. Problems from homework #4.
2. Find all integers $0 \leq c < 30$ such that $12x \equiv c \pmod{30}$ has a solution. When there are congruences, how many incongruent solutions are there?

Essential Skill 2 Know when an integer has a modular inverse and be able to find it when they exist.

1. Problems from homework #4.
2. Section 4.2 Numbers 8,9
3. **True or False:** Suppose that \hat{a} is an inverse of $a \pmod{m}$ and that \hat{b} is an inverse of $b \pmod{m}$. Then $\hat{a}\hat{b}$ is an inverse of $ab \pmod{m}$. Prove or give a counterexample.