Math 116 Number Theory
Spring 2007

Midterm 2 Practice

Name: ____________________________
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0 pts

Instructions. (0 points) *In Travis's opinion*, this exam is a rough approximation of the actual exam. He hopes that it will serve to help focus your studying for the exam. Travis did not intentionally make any mistakes in the solutions, but he is human and might have goofed on a solution. Use at your own risk.

1. *True*, *False*, or *Unknown*. Mark T if the statement is always true, mark F is the statement is ever false, and mark U if it is currently unknown by the mathematical community whether the statement is true or false.

(a) ______ If n is a composite integer other than 4 then $(n-1)! \equiv 0 \pmod{n}$.

(b) ______ A positive integer t has an inverse modulo m if and only if $\gcd(t, m) = 1$.

(c) ______ If $k > 0$ then $\phi(n) = k$ has finitely many solutions.

(d) ______ If $k > 0$ then $\sigma(n) = k$ has finitely many solutions.

(e) ______ If $k > 0$ then $\tau(n) = k$ has finitely many solutions.

(f) ______ There are infinitely many absolute pseudoprimes.

(g) ______ There are infinitely many Mersenne primes.

(h) ______ Every perfect number is even.

(i) ______ If p is prime, then $2^p - 1$ is prime.

(j) ______ If $ta \equiv tb \pmod{m}$ then $a \equiv b \pmod{m}$.

(k) ______ If $a \equiv b \pmod{m}$ and if a is relatively prime to b, then b is relatively prime to m.

(l) ______ If p is an odd prime and if $n = 2^{p-1}(2^p - 1)$ then n is a perfect number.

(m) ______ Suppose that t is the inverse of r modulo m. If r is a primitive root of m then t is also a primitive root.

(n) ______ If $m > 0$ and $0 < t < m$ then $t^{\phi(m)} \equiv 1 \pmod{m}$

2. Find a multiple of 11 that leaves remainder 2 when divided by 2, 3, 5, and 7.

3. Explain, in as much detail as it would take to be understood by a fellow student, why an integer is divisible by eleven if and only if the alternating sum of the digits is divisible by eleven.

4. Define what an *absolute pseudoprime* (i.e. Carmichael number) is.

5. Define what a *Mersenne prime* is and give three examples.

6. Compute $\phi(20!)$.

7. Find all solutions to $\phi(n) = 8$.

8. Find a perfect number larger than 10,000.

9. Find all primitive roots of 19.

10. Find a factor of $2^{17461} - 1$.