**Math 116 Number Theory** **Homework #1** **Spring 2007**
**with Travis Kelm** **Introduction** **CSU - Fresno**

This is a big league class. You must provide complete explanations for every non-obvious assertion that you make.

1. **Shapes of Numbers -**
   (a) Find the first five numbers that are square and cubic. If you can, find a formula for the $n^{\text{th}}$ such number. (Hint: How many prime factors in a square number? How many in a cubic number?)
   (b) 1 and 36 are two numbers that are triangular and square. Find three more or explain why three more do not exist.
2. **Prime Triplets -** A set of three consecutive odd numbers that are all prime is called a prime triplet. For example, $\{3, 5, 7\}$ is a prime triplet. How many prime triplets are there?
3. **Sum of Odds -** There is a formula for the sum of the first $n$ odd numbers. Find it and prove it. (Hint: You can always use algebra, but there is a lovely geometric proof.)
4. **Special Primes -** It is generally believed that there are infinitely many primes of the form $N^2 + 1$ where $N$ is a natural number. (But no one has yet been able to prove it!) How many primes are there of the form $N^2 - 1$?
5. **Odd Fibonaccis -** Find a simple formula for the sum of the first $n$ Fibonacci numbers with odd indices. That is find a formula for
$$F_1 + F_3 + F_5 + \cdots + F_{2n-1}$$
6. **Bonus Problem -** Show that every natural number that is not Fibonacci can be written as the sum of distinct, *non-consecutive* Fibonacci numbers.
   For example, 9 can be written as $5 + 3 + 1$, but 5 and 3 are consective Fibonacci numbers. So instead we write $9 = 8 + 1$ since 8 and 1 are not consecutive.
   Then explain how this fact is key for a winning strategy for Fibonacci Nim.

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