Math 116 Number Theory	**Homework #4**	**Spring 2007**	**Due 20 Feb 2007**
with Travis Kelm	**Congruences**	**CSU - Fresno**	

In your solutions you must explain what you are doing using complete sentences.

Section 3.7 - Linear Diophantine Equations

Exercise 2bc: For each of the following Diophantine equations either find all the solutions or explain why there are no integral solution.

b) $12x + 18y = 50$ c) $30x + 47y = -11$

Exercise 4: A student returning home from Europe changes his euros and Swiss francs into U.S. dollars. The exchange rates are \$1.11 for each euro and \$0.83 for each franc. If the student receives a total of \$46.26 how much of each type of currency was exchanged?

Exercise 10: At a clambake the cost of a lobster dinner is \$11 and the cost of chicken dinner is \$8. What can you conclude if the total bill is is each of the following amounts:

\$777 \$96 \$69

Exercise 13: Which combinations of pennies, dimes and quarters have a total value of \$0.99?

Not quite in book: Is it possible to have 50 coins, a mix of pennies, dimes and quarters, with a total value of \$3.00? What about \$2.00?

Section 4.1 - Intro to Congruences

Exercise 2: Determine whether each of the following pairs are congruent modulo 7:

$1, 15$	$2, 99$	$-9, 5$
$0, 42$	$-1, 8$	$-1, 699$

Exercise 4: Show that if a is an even integer, then $a^2 \equiv 0 \pmod 4$ and if a is an odd integer, then $a^2 \equiv 1 \pmod 4$.

Exercise 6: Find the least non-negative residue modulo 13 of each of the following integers:

22	1001	-100
100	-1	-1000

Exercise 8: Show that if a, b, m and n are integers such that $m > 0$, $n > 0$, $n|m$, and $a \equiv b \pmod m$ then $a \equiv b \pmod n$.

Exercise 14: Construct a multiplication table modulo 6.

Exercise 24: Give a complete system of residues modulo 13 using only odd numbers.

Exercise 33: Find the least positive residue of each of the following:

- 3^{10} modulo 11
- 2^{12} modulo 13
- 5^{16} modulo 17
- 3^{22} modulo 23
- Propose a theorem based upon the above congruences.

Exercise 34: Find the least positive residue of each of the following:

- 6! modulo 7
- 10! modulo 11
- 12! modulo 13

- 16! modulo 17
- Propose a theorem based upon the above congruences.

Section 4.2 - Linear Congruences

Exercise 2: Find all solutions to each of the following:

$$3x \equiv 2 \pmod{7} \qquad 17x \equiv 14 \pmod{21} \qquad 128x \equiv 833 \pmod{1001}$$
$$6x \equiv 3 \pmod{9} \qquad 15x \equiv 9 \pmod{25} \qquad 987x \equiv 610 \pmod{1597}$$

Exercise 6: For which integers c with $0 \leq c < 30$ does the congruence $12x \equiv c \pmod{30}$ have solutions? When there are solutions, how many incongruent solutions are there?

Exercise 10:

1. Determine the integers from 1 to 14 that have an inverse modulo 14.
2. Find the inverses of each of the integers from the previous part that have inverses.