Math 116 Number Theory **Homework #5** **Spring 2007** **Due 29 Feb 2007**
with Travis Kelm **Congruences** **CSU - Fresno**

In your solutions you must explain what you are doing using complete sentences.

Section 4.3 - The Chinese Remainder Theorem

Exercise 4abc: Find all of the solutions to each system of linear congruences

$$(a)\quad \begin{aligned} x &\equiv 4 \pmod{11} \\ x &\equiv 3 \pmod{17} \end{aligned} \qquad (b)\quad \begin{aligned} x &\equiv 1 \pmod{2} \\ x &\equiv 2 \pmod{3} \\ x &\equiv 3 \pmod{5} \end{aligned} \qquad (c)\quad \begin{aligned} x &\equiv 0 \pmod{2} \\ x &\equiv 0 \pmod{3} \\ x &\equiv 1 \pmod{5} \\ x &\equiv 6 \pmod{7} \end{aligned}$$

Exercise 12: If eggs are removed from a basket 2,3,4,5,6, and 7 at a time, there remain, respectively, 1,2,3,4,5, and 0 eggs. What is the least number of eggs that could have been in the basket?

Exercise 18: Does the system

$$\begin{aligned} x &\equiv 1 \pmod{8} \\ x &\equiv 3 \pmod{9} \\ x &\equiv 2 \pmod{12} \end{aligned}$$

have a solution? Be sure to explain why or why not.

Section 5.1 - Divisibility Tests

Exercise *: Invent your own divisibility tests for 37, 101, and 33. I will give extra points for tests that I find especially inventive or useful.

Section 6.1 - Wilson's Theorem and Fermat's Little Theorem

Exercise 4: Find the remainder when $5!25!$ is divided by 31.

Exercise 6: Find the remainder when $7 \times 8 \times 9 \times 15 \times 16 \times 17 \times 23 \times 24 \times 25 \times 43$ is divided by 11.

Exercise 12: Use Fermat's Little Theorem to find the least positive residue of 2^{10^6} modulo 7.

Exercise 16: Show that if n is composite integer other than 4, then $(n-1)! \equiv 0 \pmod{n}$.