116 - Number Theory
Spring 2007

Homework 7

Name: ______________________________
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Section 7.2 - The Sum and Number of Divisors

1. **From Class:** Show that if m and n are relatively prime, then $\tau(m \cdot n) = \tau(m) \cdot \tau(n)$.
2. **Exercise 7:** Fix a positive integer k. Show that the equation $\tau(n) = k$ has infinitely may solutions solutions.
3. **Exercise 11:** $\sigma(n)$ is the *sum* of all positive divisors of n. Find a prove a formula for the *product* of all positive divisors of n.
4. **Exercise 12:** Fix a positive integer k. Show that the if the equation $\sigma(n) = k$ has any solutions, then it has only a finite number of solutions.

Section 7.3 - Perfect Numbers and Mersenne Primes

5. **Exercise 2:** Find the seventh and eighth even perfect numbers.
6. **Exercise 4:** Find a factor of each of the following integers:

Abundant and Deficient Numbers A positive integer n is called *abundant* if $\sigma(n) > 2n$ and is called *deficient* if $\sigma(n) < 2n$. See if you can answer the following questions about such numbers. *Hint*: It is ok to use the results of other exercises (even ones that were not assigned) to help solve these problems.

7. **Exercise 5:** Find the six smallest abundant integers.
8. **Exercise 6:** Find the smallest odd abundant integer.
9. **Exercise 10:** Show that if $n = 2^{m-1}(2^m - 1)$ where $m > 0$ and $2^m - 1$ is composite, then n is abundant.
10. **Exercise 12-13:** Show that there are infinitely many even abundant numbers and infinitely many odd abundant numbers.